LECTURE - 4:

- WE SHOWED THAT IF THE FORCE IS CONSERVATIVE, THE TOTAL ENERGY IS INDEPENDENT OF THE POSITION OF THE PARTICLE.

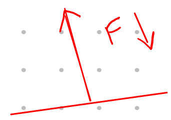
  ↳ CONSERVATION OF ENERGY

- WE ALSO LEARNE ABOUT POTENTIAL ENERGY.

$$U_b - U_a = -\int_a^b \vec{F} \cdot \vec{dr}$$

- NOW, EXAMPLE: POTENTIAL ENERGY OF A UNIFORM FORCE FIELD

  eg - UNIFORM GRAV. FIELD



$$\vec{F}_{\cdot} = mg\,\hat{k}$$

$$U_b - U_a = -\int_a^b \vec{F} \cdot \vec{dr}$$

$$= -\int_{z_a}^{z_b} (-mg)\,dz = mg(z_b - z_a)$$

ASSUMING

$U = 0$ AT GROUND

HENCE, $U(h) = mgh$, $h \rightarrow$ HEIGHT

ALSO USING, CONSERVATION LAW,

$$k_0 + U_0 = k(h) + U(h)$$

$$\Rightarrow \frac{1}{2} m v_0^2 + 0 = \frac{1}{2} m v(h)^2 + mgh$$

$$\Rightarrow v(h) = \sqrt{v_0^2 - 2gh}$$

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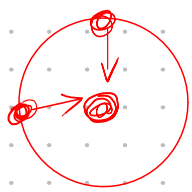
- EXAMPLE: INVERSE SQUARE FIELD



GENERALLY, CENTRAL FORCE $\vec{F} = f(r)\,\hat{r}$

eg - COULOUMB FORCE, GRAVITATIONAL FORCE

$$F \propto \frac{q_1 q_2}{r^2}, \quad F \propto \frac{Mm}{r^2}$$

BASICALLY $F \propto \frac{1}{r^2} \Rightarrow F = A\,\frac{1}{r^2}$

THE POTENTIAL ENERGY OF A PARTICLE IN CENTRAL FORCE

$$U_b - U_a = -\int_{r_a}^{r_b} \vec{F} \cdot \vec{dr} = -\int_{r_a}^{r_b} f(r)\, dr$$

SO, FOR $f(r) = A/r^2$,

$$U_b - U_a = -\int_{r_a}^{r_b} \frac{A}{r^2}\, dr$$

$$\Rightarrow U_b - U_a = \frac{A}{r_b} - \frac{A}{r_a}$$

TO OBTAIN POTENTIAL ENERGY FUNCTION, WE REPLACE r_b BY RADIAL VARIABLE r.

$$U(r) = \frac{A}{r} + \left(U_a - \frac{A}{r_a}\right)$$

$$= \frac{A}{r} + C$$

, WHERE, C = CONSTANT, HAS NO PHYSICAL MEANING

SO, WE ARE FREE TO GIVE C ANY VALUE

$C = 0$, i.e. $U(\infty) = 0$.

THEREFORE, $U(r) = \frac{A}{r}$

eg: SPRING FORCE, $\vec{F} = -k(r - r_0)\hat{r}$

CENTRAL FORCE, HENCE CONSERVATIVE,

$$U(r) - U(a) = -\int_a^r (-k)(r - r_0)\,\hat{r} \cdot dr$$

$$= +\int_a^r k(r - r_0)\, dr$$

$$= \frac{1}{2}k(r - r_0)^2 \Big|_a^r$$

SO,, $U(r) = \frac{1}{2}k(r - r_0)^2 + C$

WE CHOOSE THE POTENTIAL ENERGY TO BE ZERO AT EQUILIBRIUM,

$U(r_0) = 0$

$$U(r) = \frac{1}{2}k(r - r_0)^2$$

- Q: WHAT POTENTIAL ENERGY TELLS US ABOUT FORCE?

WE KNOW,

$$U_b - U_a = -\int_a^b \vec{F} \cdot d\vec{r}$$

BUT IT IS EASIER TO USE POTENTIAL AS MORE FUNDAMENTAL QUANTITY, BECAUSE IT IS A SCALAR.

SO CONSIDER, CHANGE IN POTENTIAL ΔU WHEN THE PARTICLE MOVES FROM x to $x+\Delta x$.

$$U(x+\Delta x) - U(x) \equiv \Delta U = -\int_x^{x+\Delta x} F(x)\, dx$$

FOR SMALL Δx, $F(x)$ is CONSTANT OVER THE RANGE OF INTEGRATION.

SO,

$$\Delta U = -F(x)(x+\Delta x - x)$$

$$= -F(x)\,\Delta x$$

$$\Rightarrow F(x) = -\frac{\Delta U}{\Delta x}$$

Force is negative derivative of potential.

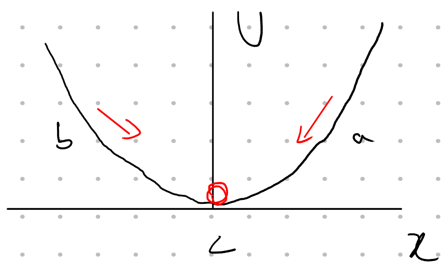
IN THE LIMIT, $\Delta x \to 0$,

$$F(x) = -\frac{dU}{dx}$$

- STABILITY AND STUFF

- POTENTIAL CAN BE USED TO ANALYZE THE STABILITY OF THE SYSTEM.

$$U = \frac{1}{2}kx^2$$



AT POINT a,

$$\frac{dU}{dx} > 0 \quad , \text{FORCE} < 0$$

AT POINT b,

$$\frac{dU}{dx} < 0 \quad , \text{FORCE} > 0$$

AT POINT c,

$$\frac{dU}{dx} = 0 \text{ . FORCE VANISHES.}$$

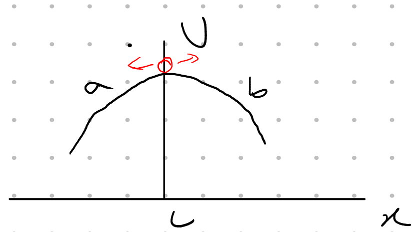
SO, THE FORCE IS DIRECTED TOWARDS ORIGIN, NO MATTER WHICH WAY THE PARTICLE IS DISPLACED.

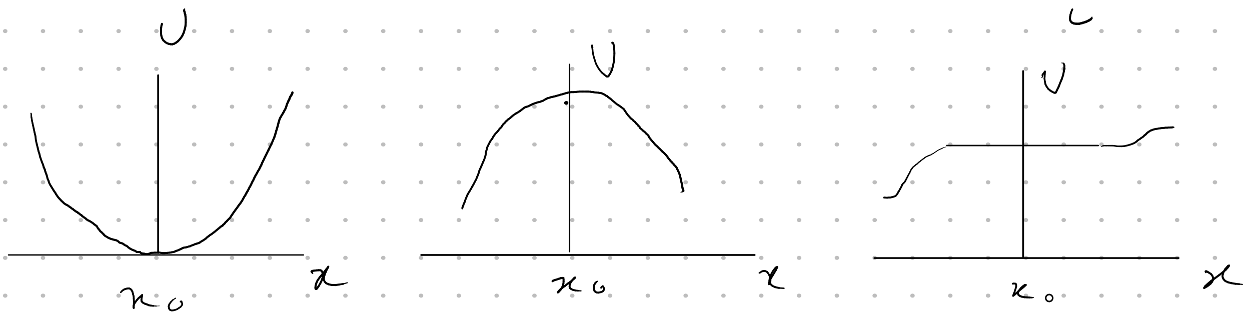
MINIMA → EQUILIBRIUM

↳ STABLE

HOWEVER, IF $\frac{dU}{dx} = 0$ AT MAXIMA, → UNSTABLE.

UNSTABLE SYSTEM.





$\frac{d^2U}{dx^2} > 0$ — STABLE

$\frac{d^2U}{dx^2} < 0$ — UNSTABLE

$\frac{d^2U}{dx^2} = 0$ — NEUTRAL

• NON-CONSERVATIVE FORCES:

- USUALLY, CONSERVATIVE FORCES ARE DISCUSSED MORE BECAUSE THEY ARE MORE IMPORTANT.
- IN SOME CASES, NON-CONSERVATIVE FORCES ARE PRESENT. SO LET'S DISCUSS THEM.
- eg: OBJECTS FALLING UNDER GRAVITY, FEELS AIR RESISTENCE AS WELL.

TOTAL FORCE IN THAT CASE –

$$\vec{F} = \vec{F}_c + \vec{F}_{nc}$$

- WORK-ENERGY THEOREM IS TRUE WHETHER OR NOT THE FORCES ARE CONSERVATIVE.

$$\therefore\ W_{ba} = \oint_a^b \vec{F}\cdot d\vec{r} = \oint_a^b \vec{F}_c\cdot d\vec{r} + \oint_a^b \vec{F}_{nc}\cdot d\vec{r}$$

$$\Rightarrow W_{ba} = \underbrace{-U_b + U_a}_{\text{POTENTIAL ENERGY ASSOCIATED WITH THE CONSERVATIVE FORCE}} + \underbrace{W_{ba}^{nc}}_{\text{WORK DONE BY THE NC FORCE.}}$$

AGAIN FROM WE THEOREM,

$$K_b - K_a = -U_b + U_a + W_{ba}^{nc}$$

$$\Rightarrow (K_b + U_b) - (K_a + U_a) = W_{ba}^{nc}$$

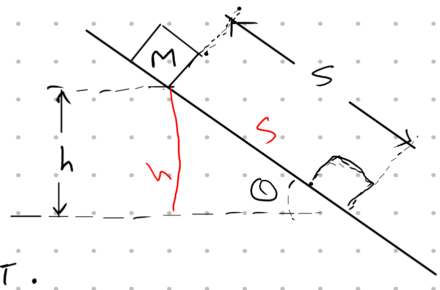
$$\Rightarrow E_b - E_a = W_{ba}^{nc} \quad //$$

SO IF NC FORCES DO NOT WORK, $E_b = E_a$ (MECHANICAL ENERGY IS CONSERVED)

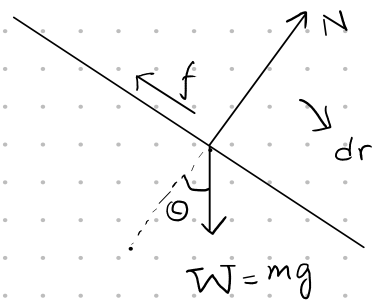
EXAMPLE: BLOCK IN AN INCLINED PLANE.

MASS $\rightarrow M$
ANGLE $\rightarrow \theta$, COEFF. FRICTION $= \mu$

TO FIND: THE SPEED OF THE BLOCK AFTER IT HAS DESCENDED THROUGH HEIGHT 'h' ASSUMING IT STARTS FROM REST.



$U_a = Mgh$	$U_b = 0$
$K_a = 0$	$K_b = \frac{1}{2}mv^2$
$E_a = Mgh$	$E_b = \frac{1}{2}mv^2$



NON-CONSERVATIVE FORCE $f = \mu N$

$$= \mu Mg\cos\theta$$

NON-CONSERVATIVE WORK DONE, $W_{ba}^{nc} = \int_a^b \vec{f}\cdot d\vec{r} = -fs = -f\frac{h}{\sin\theta}$

SO WE HAVE,

$$W_{ba}^{nc} = -\mu Mg\cos\theta \frac{h}{\sin\theta}$$

$$= -\mu \cot\theta \, Mgh$$

WHEN THERE IS A NON CONSERVATIVE FORCE INVOLVED,

$$E_b - E_a = W_{ba}^{nc}$$

$$\Rightarrow \frac{1}{2}\cancel{M}v^2 - \cancel{M}gh = -\mu\cot\theta \, \cancel{M}gh$$

$$\Rightarrow \quad v = \sqrt{2(1-\mu\cot\theta)gh}$$

NOTE: IN THE ABOVE PROBLEM, FORCES ARE CONSTANT. CAN BE DONE EASILY USING LAWS OF MOTION. HOWEVER, THIS METHOD IS USEFUL WHEN WE HAVE VARIABLE FORCES.