

Lecture - I

Mechanics of a single particle

• **Mechanics**: Science of motion of material bodies

- **Kinematics**

The motion of material bodies in terms of quantities such as DISPLACEMENT, VELOCITY etc.

$$\underline{\text{eg}} \quad \vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$
$$\vec{v} = \vec{u} + \vec{a}t$$

(basically a geometrical description)

- **Dynamics**

- It is mostly related to force.
- Study of equation of motion.

$$\underline{\text{eg}} \quad F = G \frac{mM}{r^2}$$

• Now what are classical mechanics, Quantum mechanics etc?

• In this course we shall study **dynamics of particles** -

• So now the Question -

What is a particle?

any examples? (Earth compared the Sun)

• How to study the dynamics of particles?

— NEWTON'S LAWS

① "EVERY BODY CONTINUES TO BE IN ITS STATE OF REST OR OF UNIFORM MOTION IN A STRAIGHT LINE UNLESS IT IS COMPELLED TO CHANGE THAT STATE BY EXTERNAL FORCES ACTING ON IT."

② "THE TIME RATE OF CHANGE OF MOMENTUM OF A PARTICLE IS PROPORTIONAL TO THE EXTERNAL FORCE AND IS IN THE DIRECTION OF THE FORCE."

③ "TO EVERY ACTION, THERE IS ALWAYS AN EQUAL AND OPPOSITE REACTION."

Inertia?

Force?

• Second law's meaning of Force

Force $\rightarrow$ Change of momentum/time
($\vec{p} = m \times \vec{v}$)

$$\text{So, } \vec{F} \propto \frac{d\vec{p}}{dt}$$

$$F = k \frac{d\vec{p}}{dt}$$

$$= k \frac{d}{dt} (m\vec{v})$$

$$= k m \left(\frac{d\vec{v}}{dt} \right)$$

$$\Rightarrow \vec{F} = k m \vec{a}$$

$\hookrightarrow$ constant of proportionality

$$\vec{F} \Rightarrow (1 \text{ kg mass}) \rightarrow (1 \text{ m/s}^2 \text{ accel}^n)$$

$\hookrightarrow 1 \text{ N Force}$

$$1 = k \times 1 \times 1$$

$$k = 1 \text{ unit}$$

$$\boxed{\vec{F} = m \vec{a}}$$

• From 2nd law, When $\vec{F} = 0$, $m \vec{a} = 0$

$$\frac{d}{dt} (m\vec{v}) = 0$$
$$\boxed{m\vec{v} = \text{const.}}$$

(2)

• What if there are more forces present in the system?

- If more forces are present, $F_1, F_2, \dots, F_n$, acting on the particle, then the total effect in motion produced by these forces can be looked upon as produced by a single force

- Principle of Superposition.

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n \quad (\text{vector sum})$$

• What about the third law now?

- applies to two isolated particles exerting forces on each other.
(When ext. forces are absent.)

- Action and reaction = mutual interaction.

$$\begin{aligned} - \vec{F}_1 &= -\vec{F}_2 \\ &= \end{aligned}$$

• Where can we apply these laws?

?

• Example - to measure masses.

$$\Rightarrow \vec{F} = m \vec{a} \text{ to measure mass.}$$

= Let the same force action on m_1 and m_2 produces acc'n $\vec{a}_1$ and $\vec{a}_2$.

$$m_1 \vec{a}_1 = m_2 \vec{a}_2$$

Numerically

$$m_1 a_1 = m_2 a_2$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{a_2}{a_1}$$

↳ consider to be unit mass
 $m_2 = 1 \text{ kg}$

measure a_2 and a_1

On this way you can measure the mass of m_1 .

- masses measured in this way are called

Inertial mass

- Gravitational mass.

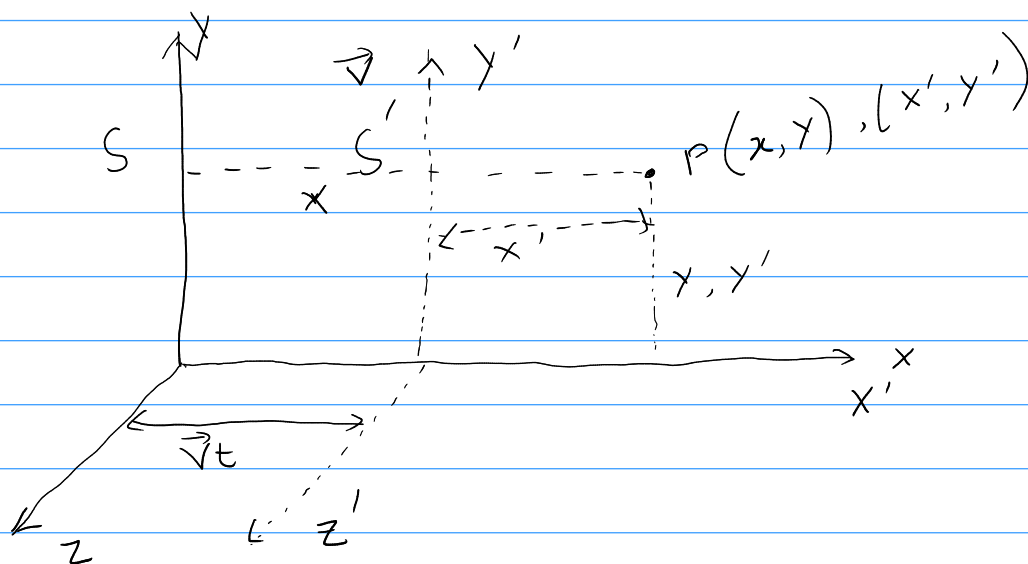
$$\vec{W} = m \vec{g} \rightarrow \text{acc'n due to gravity } (g \approx 9.8 \text{ m/s}^2)$$

↳ gravitational mass

Inertial mass $\sim$ gravitational mass (10^{10})
Weak equivalence principle.

- Inertial frame of reference:

Coordinate system.



- How are these coordinates related?

- By Galilean transformation eq^{ns}.

$$x' = x - vt \quad \text{--- (1)}$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

- How to check the validity of Newton's laws in this system?

$$\begin{aligned} \textcircled{1} \Rightarrow \quad x' &= x - vt \\ \frac{dx'}{dt} &= \frac{dx}{dt} - v \\ \frac{d^2x'}{dt^2} &= \frac{d^2x}{dt^2} \quad // \end{aligned} \quad \left| \quad \begin{aligned} m \frac{d^2x'}{dt^2} &= m \frac{d^2x}{dt^2} \\ \Rightarrow m \vec{a}' &= m \vec{a} \end{aligned} \right.$$

Newton's laws are same in all inertial reference frames.